

MATHEMATICS HSSC-II

Time allowed: 2:35 Hours

Total Marks Sections B and C: 80

SECTION – B (Marks 48)

Q. 2 Solve the following Questions.

(12 x 4 = 48)

(Use of graph paper is not necessary. Candidates can make their own grid on answer book)

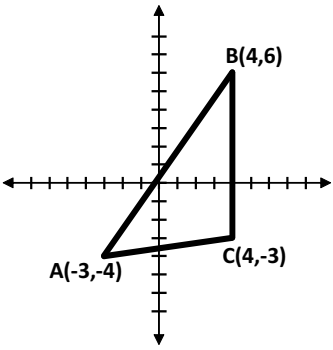
(i)	If $f(x) = \frac{3x+2}{2x-1}$, find $f^{-1}(x)$ and also show that $f^{-1}(f(x)) = x$	04	OR	Find $\frac{dy}{dx}$ if $2y^3 - 3xy^2 + 2x^2y + 5x = 6$ Also find the value of $\frac{dy}{dx}$ at (1,1)	04
(ii)	Find the derivative of $y = (2\sqrt{x} + 2)(x - \sqrt{x})$	04	OR	Discuss the continuity of the function at $x = 1$ $f(x) = \begin{cases} 3x-1 & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ 2x & \text{if } x > 1 \end{cases}$	04
(iii)	If $y = \cot(q \cot^{-1}x)$ then show that $(1+x^2)y_1 - q(1+y^2) = 0$	04	OR	Evaluate $\lim_{\theta \rightarrow 0} \frac{\sec\theta - 1}{\theta \sec\theta}$	04
(iv)	Examine the function $x^3 - 6x^2 + 9x + 3$ for extreme values.	04	OR	Use the differential to approximate value of $\sin 61^\circ$	04
(v)	Find the area bounded by the curve $y = x^3 - 9x$ and the x-axis.	04	OR	Find the area of the region bounded by $10x^2 - xy - 21y^2 = 0$ and $x + y + 1 = 0$	04
(vi)	Solve the differential equation $\frac{dy}{dx} = \frac{3}{4}x^3 + x - 3$ if $y = 0$, when $x = 2$	04	OR	Find the point P on the joint of A(1,4) and B(5,6) that is twice as far from A as B is from A and lies on opposite side of A as B does.	04
(vii)	The length of perpendicular from origin to the line is 8 unit and angle of inclination is 30° . Find the slope and y-intercept of the line.	04	OR	Integrate $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$	04
(viii)	Find the equation of a circle passing through the point (-2, -5) and touching the line $3x + 4y - 24 = 0$ at the point (4,3)	04	OR	Graph the feasible region of the following system of linear inequalities by shading and find the corner points $3x + 2y \geq 6, x + y \leq 4, x \geq 0, y \geq 0$	04
(ix)	Graph the feasible region of the following system of linear inequalities by shading and find the corner points. $5x + 7y \leq 35, x - 2y \leq 4, x \geq 0, y \geq 0$	04	OR	Find the equation of parabola having Focus (-3,4) and directrix $3x + 2y - 3 = 0$	04
(x)	Prove that altitudes of a triangle are concurrent (by vector method).	04	OR	Find the value of C, when the line $5x + 2y + C = 0$ will touch the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$	04
(xi)	Find the points of intersection to the two conics $\frac{x^2}{18} + \frac{y^2}{8} = 1$ and $\frac{x^2}{3} - \frac{y^2}{3} = 1$	04	OR	If $\vec{u} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{v} = \hat{i} + 4\hat{j} + 3\hat{k}$ and $\vec{w} = \hat{i} + 7\hat{j} + \lambda\hat{k}$ represent the side of a triangle. Find the value of λ	04
(xii)	Find the equation of an ellipse with foci $(\pm\sqrt{5}, 0)$ and passing through the point $(\frac{3}{2}, \sqrt{3})$	04	OR	Find the moment about (1, 1, 1) of each of the concurrent forces $\hat{i} - 2\hat{j}$, $3\hat{i} + 2\hat{j} - \hat{k}$ and $5\hat{j} + 2\hat{k}$ where P(2, 0, 1) is the point of concurrency.	04

SECTION – C (Marks 32)

Note: Solve the following Questions.

(4 x 8 = 32)

(Use of graph paper is not necessary. Candidates can make their own grid on answer book)

Q.3	Let $f(x) = \begin{cases} px+2 & \text{if } 0 \leq x < 2 \\ 7-qx & \text{if } 2 \leq x < 4 \\ 2x+1 & \text{if } 4 \leq x < 6 \end{cases}$	Find the values of p and q such that $f(x)$ is continuous at $x = 2$, and $x = 4$, sketch the graph of $f(x)$ after finding the values of p and q .	08	OR	A box with a square base and open top is to have a volume 32 cubic dm. Find the dimensions of the box that will require the least material.	08
Q.4	Evaluate $\int \frac{2x^2 - x - 7}{(x+2)^2(x^2 + 2x + 5)} dx$		08	OR	Find the equation of tangent of an ellipse $\frac{x^2}{128} + \frac{y^2}{18} = 1$ which are parallel to the line $3x + 8y + 1 = 0$ Also find the point of contact.	08
Q.5	The diagram shows a triangle ΔABC where $A(-3,-4)$, $B(4,6)$ and $C(4,-3)$  a. Write the equation of sides \overline{AB} and \overline{AC} . b. Find interior angle, $\angle A$. c. Find the area of ΔABC d. Find the perpendicular distance from point $C(4,-3)$ to line \overline{AB}		08	OR	A factory produces two items ceiling lights and ceiling fans by using two machines A and B. Machine A has at most 120 hours available and machine B has maximum 144 hours available. Manufacturing of a ceiling light requires 5 hours in machine A and 4 hours in machine B, while manufacturing of a ceiling fan requires 4 hours in machine A and 8 hours in machine B. If the factory gets a profit of Rs 50 per ceiling light and Rs 80 per ceiling fan, how many ceiling lights and ceiling fans should be manufactured to get maximum profit?	08
Q.6	Find the centre, foci, eccentricity, vertices and equation of directrices of the conic $9x^2 - y^2 - 12x - 2y + 2 = 0$		08	OR	Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos^2 \theta \cot^2 \theta d\theta$	08

— (B) —